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Mass spectrum of the temperature dependent Bethe-Salpeter equation for composites of quarks with a Coulomb plus a linear kernel

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Abstract. We assume that high-energy nucleus-nucleus interactions give rise to a hot and dense plasma comprising quarks and gluons due to successive nucleon-nucleon collisions. We treat this plasma as an ideal fluid at temperature T prior to its eventual particlization, and attempt a microscopic description of meson-formation by using finite temperature propagators for q and $\bar{q}$ in a Bethe-Salpeter equation with a Coulomb plus a linear kernel. The equation thus obtained is reduced to a Schrodinger-like equation which then yields, for $0 < T < 86$ MeV, the bound state masses for different states of $b\bar{b}$ and $c\bar{c}$, which are in agreement with the experimental values for these states. The effects of the temperature of the plasma on the meson masses show up only when the temperature exceeds 86 MeV, pointing to the probable reason for the successes of the earlier models which did not explicitly take the temperature of the plasma into account.

PACS. 12.40.-y Other models for strong interactions

1 Introduction

In the current 'standard' model, hadrons are considered to be composites of more elementary entities (quarks). The theoretical base of the model is provided by the belief in an underlying gauge theory of hadrons on the one hand, and certain phenomenological features of quarks (for example, their non-observability in the free state) on the other. Within this framework, an enormous amount of theoretical activity has taken place in the last twenty years, particularly after the experimental discoveries of the Charmonium, the Upsilonium, etc. families. These attempts to explain the observed features of hadron spectra have differed in the choice of description (nonrelativistic or relativistic), in the choice of the inter-quark potential, and in whether the spin of the quarks is or is not taken into account. Excellent descriptions of these attempts can be found, for example, in the review articles by Mukherjee et al. [1] and by Mitra [2].

The initial thrust of activity in this field was concerned with the reproduction of the observed masses of the meson families through the quantum mechanics of the bound states of appropriate quarks interacting via suitably chosen potentials. As is well known [1, 2], the choice of the inter-quark potential is dictated by the twin requirements of asymptotic freedom and infra-red slavery, which are met by potentials of the form

$$V(r) \sim \alpha_s/r - \lambda r^{\varepsilon} - C, \tag{1}$$

where $r = |\boldsymbol{r}|$ is the inter-quark distance, α_s and λ are the coupling constants corresponding to the 'Coulomb' and the 'confining' parts of the potential respectively, ε is a positive rational number and C is a constant. While choices for ε have varied from 0.1 to 2.0, the most popular have been 1.0 and 2.0 (the 'linear' and the 'harmonic oscillator' potentials). A wide variety of these models have been successful to varying degrees of precision in accounting for the experimental meson masses. A feature of the potentials given by (1) is that, whereas the Coulomb part corresponds to a one-gluon exchange potential, the confining part does not follow directly from field theory. While the problem of a satisfactory explanation of the origin of this part of the potential remains, the thrust of activity in the field has shifted to the nature of matter which "particlizes" into the observed mesons. Specifically, it has been speculated that in high-energy nucleus-nucleus interactions, nuclear matter might be compressed or heated as a result of successive nucleon-nucleon collisions to give rise to a hot and dense plasma comprising quarks and gluons. Thus, the question: Does quark-gluon plasma exist? It is clear that the shift in the focus of inquiry mandates a thermodynamic and statistical approach [3]. One of the basic issues sought to be settled is: What is the signature of such a state of matter, if it exists?

It seems to us that the explanation of the observed meson masses assuming the existence of quarks interacting via a potential has hitherto been considered as an alternative to the explanation based on statistical and thermodynamic considerations. Most likely it was Fermi [4] and Landau [5] who initiated this latter approach which, as discussed by Grammer et al. [6], suggests that a process such as e$^+$-e$^-$ annihilation leads to the creation of hadronic matter which may be treated as an ideal fluid at temperature T prior to its eventual "particlization". While Grammer et al. have come up with some interesting conjectures regarding the gross features of meson phenomenology, they have also remarked that the statistical and thermodynamic description is not likely to teach us very much about the fundamental properties of the hadronic constituents at a deeper than macroscopic level. Similar thinking seems to prevail in the recent concern about the existence of quark-gluon plasma [3]. This need not be so. Specifically, we note that the observed meson masses have not been obtained on the basis of a dynamical description of the quark-gluon plasma. In particular, the temperature of the plasma plays no explicit role in the detailed theoretical framework in which fits have been obtained to the observed meson masses. One could argue that the value of the coupling constants used in (1) are such that they implicitly incorporate the effects of temperature. However, this is a conjecture. It is therefore interesting to investigate the effect of the explicit incorporation of temperature in the description of the "particlization" of the quark-gluon plasma. This is the problem to which we address ourselves here.

The need for such an investigation also stems from the observation that, despite an increasing appreciation in recent years of the crucial role played by temperature in the "birth" of various species of particles [7], a detailed account of the underlying mechanism has been lacking. We note that while Hagedorn [8], in a series of papers in the sixties, did consider the role played by temperature in determining the hadronic mass spectra, his philosophy ("nuclear democracy") is quite different from the one followed by us ("hadrons as composites of quarks"). Since temperature is a statistical concept, it is obvious that our investigation is tantamount to attempting a microscopic description of particlization in a macroscopic background. We are enabled to do so through the use of temperature-dependent Green's functions (not known at the time [4] and [5] were written) for the constituents of the hadrons. It should be recalled that such a function describes the motion of one particle in a many-particle system; in addition, it carries all statistical mechanical information because it is an expectation value in the grand canonical ensemble [9].

2 Dynamics of particlization

As discussed by Grammer et al. [6], one may assume that a process such as e$^+$-e$^-$ annihilation, at sufficiently high energy, leads to the creation of quark-gluon plasma in a volume the dimension of which is much smaller than the characteristic length of strong interaction forces. Following Fermi [4] and Landau [5], we treat this matter as an ideal fluid in thermal equilibrium at temperature T. What leads to the formation of the mesons are the high-energy collisions between the quarks of appropriate flavours. For the motion of a q or a $\bar{q}$ within the plasma, we invoke the concept of a finite-temperature propagator. Assuming now a suitable potential between a q and a $\bar{q}$, and making use of the special interaction representation in terms of temperature, as in e.g. Kirzhnits [9] or Fetter and Walecka [10], one can derive a temperature-generalized Bethe-Salpeter (BS) equation for the pairing amplitude for $q\bar{q}$ in the manner of Gell-Mann and Low [11]. Such an exercise has been carried out for the Coulomb problem by Pande [12], and it has been established that the resulting equation is identical to the equation obtained earlier by Malik et al. [13] by using the Matsubara prescription in the $T = 0$ equation [14]. Naturally, therefore, we follow the latter course here.

The $T = 0$ momentum-space BS equation for the bound state of $q\bar{q}$ is [16]

$$\chi_{\mathrm{p}}(q) = -\int d^4k S_{\mathrm{F}}'^{(1)}(p_1) S_{\mathrm{F}}'^{(2)}(p_2) G(P,q,k)\chi_{\mathrm{p}}(k), \quad (2)$$

where p_1 and p_2 are the final 4-momenta, p_1' and p_2' are the initial 4-momenta, and k and q are the initial and final relative 4-momenta respectively. Confining ourselves to the equal-mass case, these variables are given by

$$k = (p_1' - p_2')/2, \quad q = (p_1 - p_2)/2, \quad P = p_1 + p_2 = p_1' + p_2'.$$

Taking recourse to the ladder approximation, we replace the exact fermion propagators $S_{\mathrm{F}}'^{(1)}$ and $S_{\mathrm{F}}'^{(2)}$ in (2) by free fermion propagators and the interaction function G by its lowest order value:

$$G(P,q,k) \simeq G_0(q,k) = [1/(2\pi)^4] F_{12} \gamma_\mu^{(1)} \gamma_\mu^{(2)} \langle q|V_{12}|k\rangle, \quad (3)$$

where the color factor F_{12} has the value $-4/3$ for the $q\bar{q}$ system. We now go over to the center of mass frame of a $q\bar{q}$ pair so that $P = (\mathbf{0}, iM)$. Substituting (3) into (2) and carrying out the spin reduction of the resulting equation by the method of Gordon [16], we obtain

$$D(q_0, \boldsymbol{q})\chi_{\mathrm{p}}(q) = -[F_{12}/(2\pi)^4] \int d^4k I(q,k)\chi_{\mathrm{p}}(k), \quad (4)$$

where

$$\begin{aligned} D(q_0, \boldsymbol{q}) &= [(q_0 + \boldsymbol{M}/2)^2 - w^2][(q_0 - \boldsymbol{M}/2)^2 - w^2], \quad (5)\\ w &= (m^2 + \boldsymbol{q}^2)^{1/2}, \\ I(q,k) &= [P^2 - (q+k)^2 + \sigma_{\mu\nu}^{(1)}\sigma_{\mu\lambda}^{(2)}(q-k)_\nu(q-k)_\lambda \\ &\quad - i\{\sigma_{\mu\nu}^{(1)} - \sigma_{\mu\nu}^{(2)}\}P_\mu(q-k)_\nu \\ &\quad - 2i\{\sigma_{\mu\nu}^{(1)} + \sigma_{\mu\nu}^{(2)}\}q_\mu k_\nu\}\langle q|V_{12}|k\rangle. \quad (6) \end{aligned}$$

We note that the spin-reduction of the equation (Gordon reduction) is achieved by assuming the constituents of the bound states to be "on-shell". This is evidently open to objection in view of the bound state nature of the problem. However, the elegant and compact manner in which the spin-spin, spin-orbit, tensor and "contact" terms now appear (see (16)) is a rather pleasant feature of this approach, which is to be constrasted with the highly unwieldly decomposition that the more familiar method of separation into large and small components would lead to. While we refer to [17] for a more detailed discussion of the Gordon decomposition, we note that an important consideration for us was also to study the temperature-generalized version of the model within the same approximation scheme as was used in the original $T = 0$ model [18].

Equation (4) is reduced to a 3-dimensional equation by making the instantaneous approximation (IA), which consists of two steps. First, one sets $k_0 = q_0$. This causes $I(q, k)$ to become of the form $c_0 + c_1 q_0 + c_2 q_0^2$, where c_0, etc. are functions of $\boldsymbol{q}$ and $\boldsymbol{k}$. Again, in order to study the temperature-generalized model within the same set of approximations as adopted in [18], we also neglect the q_0 and q_0^2 terms. Then the r.h.s. of (4) is a function of $\boldsymbol{q}$ alone. Consistency now demands that we equate the l.h.s. of the equation also to a function of $\boldsymbol{q}$. Thus, the second step in the IA consists of setting

$$D(q_0, \boldsymbol{q})\chi_{\rm p}(q) = \psi(\boldsymbol{q}), \tag{7}$$

whence (4) becomes

$$\psi(\boldsymbol{q}) = -[F_{12}/(2\pi)^4] \int d^3\boldsymbol{k}\, I(\boldsymbol{q}, \boldsymbol{k})\, \psi(\boldsymbol{k})\, J(\boldsymbol{k}), \tag{8}$$

where

$$\begin{aligned} I(\boldsymbol{q}, \boldsymbol{k}) = [&-M^2 - (\boldsymbol{q}+\boldsymbol{k})^2 - 2i\{\sigma_{ij}^{(1)} + \sigma_{ij}^{(2)}\}q_i k_j \\ &+ \sigma_{ij}^{(1)}\sigma_{il}^{(2)}(\boldsymbol{q}-\boldsymbol{k})_j(\boldsymbol{q}-\boldsymbol{k})_l \\ &- M^2(\boldsymbol{q}^2 - \boldsymbol{k}^2)/(2m^2)]\langle \boldsymbol{q}|V_{12}|\boldsymbol{k}\rangle, \end{aligned} \tag{9}$$

and

$$\begin{aligned} J(\boldsymbol{k}) &= \int dk_0/D(k_0, \boldsymbol{k}) \\ &\equiv (1/d)\,\Big[\int (k_0 + M)dk_0/\{(k_0 + M/2)^2 - w^2 + i\varepsilon\} \\ &\quad - \int (k_0 - M)dk_0/\{(k_0 - M/2)^2 - w^2 + i\varepsilon\}\Big] \quad (10) \\ &= (M/d) \int dq_0/\{q_0^2 - w^2 + i\varepsilon\} \end{aligned} \tag{11}$$

with

$$d = 2M[M^2/4 - w^2].$$

We note that (11) is obtained after appropriately redefining the variables of integration in (10) and noting that in both integrands, terms which are odd in the variable of integration yield vanishing contributions as the range of integration is symmetric. The location of poles of the integrand in (11) allows one to carry out the familiar Wick rotation, which simply changes the path of integration from $-\infty$ to $+\infty$ to $-i\infty$ to $+i\infty$. One then sets $q_0 = iq_4$ to obtain a Euclidean integral of the form of a Mellin transform. Thus

$$J(\boldsymbol{k}) = -i\pi/[2(m^2 + \boldsymbol{k})^{1/2}\{M^2/4 - (m^2 + \boldsymbol{k}^2)\}]. \tag{12}$$

The temperature-generalization of (2) is brought about by applying the Matsubara prescription [14] to (11); see also [15].

$$k_0 = (2n+1)/\pi(-i\beta)$$
$$(\beta = 1/k_{\rm B}T,\ k_{\rm B} \text{ is the Boltzmann constant})$$

with the replacement

$$\int_{-\infty}^{\infty} dk_0 \to (2\pi/-i\beta) \sum_{n=-\infty}^{\infty}.$$

This yields

$$J_\beta(\boldsymbol{k}) = -\frac{i\pi\, \tanh[(\beta/2)(m^2 + \boldsymbol{k}^2)^{1/2}]}{2(m^2 + \boldsymbol{k}^2)^{1/2}[M^2/4 - (m^2 + \boldsymbol{k}^2)]} \tag{13}$$

which, in the limit $T \to 0$, reduces to the standard zero temperature result given in (12).

It is remarkable that for the bound state of $b\bar{b}$, the value of the tanh function in (13) differs little from 1 for $0 \le T \le 100$ MeV. In this temperature regime, therefore, the presence of $\boldsymbol{k}^2$ in the argument of the tanh function makes practically no difference to its value, and it is an excellent approximation to set

$$\tanh[(\beta/2)(m^2 + \boldsymbol{k}^2)^{1/2}] \simeq \tanh[\beta m/2]. \tag{14}$$

We now need to choose $\langle \boldsymbol{q}|V_{12}|\boldsymbol{k}\rangle$ for the inter-quark potential. The following choice is based on the work of Eichten et al. [19]:

$$\begin{aligned} \langle \boldsymbol{q}|V_{12}|\boldsymbol{k}\rangle = &\ 4\pi\alpha_{\rm s}/(\boldsymbol{q}-\boldsymbol{k})^2 + 8\pi\lambda/(\boldsymbol{q}-\boldsymbol{k})^4 \\ &- (2\pi)^3 C\delta^3(\boldsymbol{q}-\boldsymbol{k})/(m^2), \end{aligned} \tag{15}$$

where, as discussed in [18], the constant term has been added to simulate a term that arises in the Gordon reduction of the original equation. Substituting (13), (14) and (15) into (8) and transforming the resulting equation to coordinate space, we obtain

$$\begin{aligned} &[-4m - (16\lambda'/3)r + 16\alpha_{\rm s}'/(3r) - 16C'/(3m^2)]\nabla^2\chi(\boldsymbol{r}) \\ &= (8/3)[(2 + M^2/2m^2)(\alpha_{\rm s}'/r^2 + \lambda')]\frac{(\boldsymbol{r}\cdot\boldsymbol{\nabla})}{r}\chi(\boldsymbol{r}) \\ &\quad + (4\alpha_{\rm s}'/3)[M^2/r + 4\pi\delta^3(\boldsymbol{r})(1 - \boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 + M^2/2m^2) \\ &\qquad - (1/r^3)\{S_{12} + 4\boldsymbol{L}\cdot\boldsymbol{S}\}]\chi(\boldsymbol{r}) \\ &\quad + (4\lambda'/3)[-M^2 r + \{2(1 + M^2/2m^2) - S_{12}/3 - 4\boldsymbol{L}\cdot\boldsymbol{S} \\ &\qquad - (4/3)(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\}/r]\chi(\boldsymbol{r}) \\ &\quad + [m(M^2 - 4m^2) - 4C'M^2/(3m^2)]\chi(\boldsymbol{r}), \end{aligned} \tag{16}$$

where $\alpha_{\rm s}' = \alpha_{\rm s}t$, $\lambda' = \lambda t$, $C' = Ct$ and $t = \tanh(m\beta/2)$.

Table 1. a. Theoretical Mass of the n^{2S+1} Lj states of $b\bar{b}$ as a function of temperature, with $m_b = 4.955$ GeV, $\alpha_S = 0.6$, $\lambda = 0.089$ GeV^2, $C = -0.112$ GeV^3

State $n^{2S+1}L_j$	M exp.	$T =$ 1e12	3.165e12	1e13	3.165e13	1e14	3.165e14	1e15	3.165e15	1e16	3.165e16
		M (theoretical)									
1^3S_1	9.460	9.465	9.465	9.469	9.659	9.919	9.951	9.936	9.924	9.917	9.914
2^3S_1	10.023	10.022	10.022	10.023	10.074	10.078	10.011	9.961	9.935	9.922	9.916
3^3S_1	10.355	10.341	10.341	10.341	10.315	10.187	10.056	9.981	9.944	9.926	9.918
4^3S_1	10.580	10.597	10.597	10.596	10.510	10.278	10.096	9.999	9.952	9.930	9.919
5^3S_1	10.865	10.825	10.825	10.822	10.685	10.361	10.132	10.015	9.959	9.933	9.921
6^3S_1	11.020	11.037	11.037	11.033	10.848	10.438	10.165	10.030	9.966	9.936	9.922
1^1S_0	9.370	9.389	9.389	9.393	9.588	9.908	9.951	9.936	9.924	9.917	9.914
2^1S_0	9.963	9.990	9.990	9.991	10.046	10.072	10.010	9.961	9.935	9.922	9.916
3^1S_0	10.298	10.318	10.318	10.318	10.294	10.182	10.056	9.981	9.944	9.926	9.918
4^1S_0	10.573	10.578	10.578	10.576	10.493	10.274	10.095	9.999	9.952	9.930	9.919
1^3P_0	9.859	9.004	9.004	9.100	9.535	10.039	9.992	9.952	9.931	9.920	9.915
2^3P_0	10.232	9.972	9.972	9.975	10.054	10.151	10.039	9.973	9.940	9.924	9.917
1^3P_1	9.891	9.852	9.852	9.855	9.988	10.043	9.992	9.952	9.931	9.920	9.915
2^3P_2	10.255	10.194	10.194	10.196	10.236	10.154	10.040	9.973	9.940	9.924	9.917
1^3P_2	9.913	9.920	9.920	9.923	10.012	10.045	9.992	9.952	9.931	9.920	9.915
2^3P_2	10.268	10.248	10.248	10.248	10.256	10.156	10.040	9.973	9.940	9.924	9.917

Note that the total load of temperature dependence is carried by the factor $\tanh(m\beta/2)$ multiplying α_s, λ and C. It is easy to see that this feature is independent of the potential used for confinement. We conclude this section by drawing attention to an important fact: The $T = 0$ equation corresponding to (16) is usually made tractable by the selective use of the approximation $M = 2m$; see, e.g., [17, 18]. In a recent work [25], we have also obtained and studied the equivalent of (16) by making the said approximation. Interestingly enough, while the two approaches lead to generally adequate fits to the observed masses, they imply quite different scenarios for the physics of the problem. This is discussed further in the last section.

3 Results and discussion

We have, using the fourth order variable-step Runge-Kutta-Merson method, numerically explored solutions of (16) over an extensive domain of parameter space after incorporating the boundary conditions that ensure the proper behavior of the solution at the origin. The bound-state energy is obtained to within an accuracy ΔE by requiring that for two values of the energy E_a and E_b differing by ΔE, the corresponding solutions $u(r)$ in the limit of large r tend to $+\infty$ and $-\infty$ respectively. The value of n characterising the bound-state is determined by the number of nodes in the solution. The same set of parameters

$$\alpha_s = 0.6, \quad \lambda = 0.089\ (\text{GeV})^2, \quad C = -0.112\ (\text{GeV})^3,$$

give a reasonably good fit to both the $b\bar{b}$ and the $c\bar{c}$ states, for any temperature in the range $0 \le T \le Tu$ (Tu is about 86 MeV for $b\bar{b}$, and slightly lower for $c\bar{c}$); thus, the bound-state energies are not dependent on temperature in this range. The masses of the b and c quarks have been taken to be 4.955 GeV and 1.52 GeV respectively.

For $T < Tu$, therefore, temperature plays the role of a dormant variable. For $T > Tu$, temperature becomes active and the meson masses depend on its value, just as they depend on the values of α_s, λ and C. We list, in Tables 1(a) and 1(b), the masses of 26 $b\bar{b}$ and $c\bar{c}$ states (22 of these have been experimentally determined [20] while the rest are theoretical predicted in the Eichten and Quigg model [21]) together with the corresponding theoretical masses calculated by us to an accuracy of ± 0.001 GeV. Our calculated values for the $^3S_1$ states of the $b\bar{b}$ and $c\bar{c}$ systems are also plotted as functions of temperature in Figs. 1 and 2 respectively to display their dependence on temperature. The Tables cover both regions – the one in which temperature is a dormant variable and the other in which it is an active variable. One would naturally expect that in the former region, our model should give essentially the same results as the $T = 0$ model of which it is a temperature-generalized extension. This is indeed so not only for each of the 14 states that the $T = 0$ model of Arafah et al. dealt with, but also for the 4 states predicted in [21]. We note that the fits to a few of the $c\bar{c}$ states do not have the same accuracy as for the $b\bar{b}$ states – this feature is a carry over of the original $T = 0$ model that we have temperature-generalized. This seems to indicate that the potential that binds the $c\bar{c}$ states is somewhat different from the one that binds the $b\bar{b}$ states, as has been suggested in [22].

In the region in which the mass of a bound state varies with temperature, the notion of mass as a fundamental attribute of a particle becomes untenable. What seems interesting is that our model gives an estimate of the temperature (Tu) at which a particle loses this fundamental attribute.

Table 1. b. Theoretical Mass of the n^{2S+1} Lj states of $c\bar{c}$ as a function of temperature, with $m_c = 1.52$ GeV, $\alpha_S = 0.6$, $\lambda = 0.089$ GeV^2, $C = -0.112$ GeV^3

State $n^{2S+1}L_j$	M exp.	T = 1e12	3.165e12	1e13	3.165e13	1e14	3.165e14	1e15	3.165e15	1e16	3.165e16
		M (theoretical)									
1^3S_1	3.097	3.106	3.107	3.135	3.160	3.119	3.081	3.060	3.050	3.045	3.043
2^3S_1	3.686	3.627	3.625	3.547	3.359	3.202	3.117	3.077	3.057	3.048	3.044
3^3S_1	4.040	4.117	4.112	3.920	3.529	3.272	3.147	3.090	3.063	3.051	3.045
4^3S_1	4.415	4.615	4.607	4.289	3.689	3.335	3.147	3.102	3.069	3.054	3.047
1^1S_0	2.979	3.031	3.032	3.067	3.142	3.116	3.081	3.060	3.050	3.045	3.043
1^3P_0	3.415	2.796	2.801	2.950	3.276	3.169	3.103	3.070	3.054	3.047	3.043
1^3P_1	3.510	3.305	3.307	3.357	3.289	3.170	3.103	3.070	3.054	3.047	3.043
1^3P_2	3.556	3.424	3.424	3.418	3.297	3.172	3.103	3.070	3.054	3.047	3.046
1^3D_1	3.770	3.432	3.438	3.586	3.394	3.213	3.121	3.078	3.058	3.049	3.045
2^3D_1	4.159	3.796	3.796	3.881	3.555	3.280	3.150	3.091	3.064	3.051	3.046

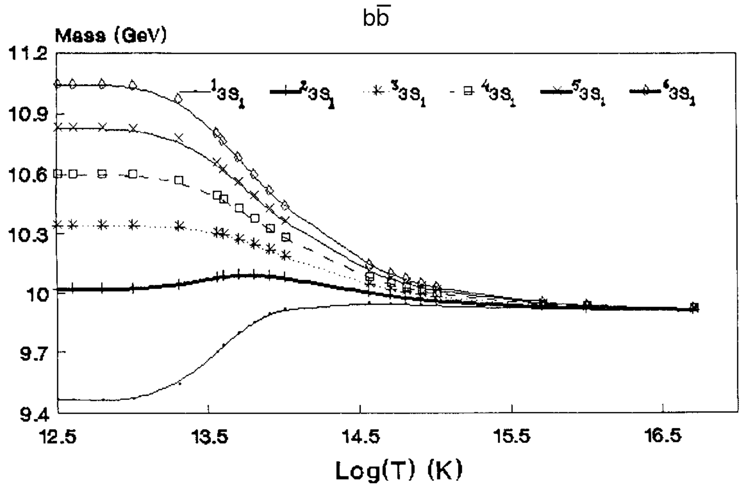

Fig. 1. Variation of theoretical mass of n^3S_1 states of $b\bar{b}$ with logarithm of temperature

In the limit of high temperatures, the bound-state energies tend to collapse to the values $2m_b$ or $2m_c$ for the $b\bar{b}$ and $c\bar{c}$ systems respectively. This is in agreement with the high temperature behaviour of the system noted earlier i.e. the effetive coupling constants go to zero and the systems become deconfined in the large temperature limit. The fact that our numerical results agree with theoretical analysis in the small and large T limits gives greater credence to our results at intermediate values of T.

Let us now compare the above results with the results obtained by selectively using the $M = 2m$ approximation [25]. The parameters which give a resonably good fit for the $b\bar{b}$ family are: $m_b = 4.764$ GeV, $\alpha_s = 7.8 \times 10^{-2}$, $\lambda = 9.9 \times 10^{-5}$ GeV^2, $C = -1.684$ GeV^3 and $T = 20.89$ MeV. With the same values of α_s and λ, the fits for the $c\bar{c}$ family are obtained with $m_c = 1.565$ GeV, $C = -0.05$ GeV^3 and $T = 5.64$ MeV. The most significant difference between [25] and the present investigation is that, in the former, temperature never plays the role of a dormant variable. Also, they lead to different deconfinement temperatures. There is thus a need to put the two approaches to more stringent tests. We are currently looking into this aspect.

Before we conclude, it is pertinent to ask: If the formation of the mesons take place in the medium comprising the quark-gluon plasma, as we have assumed in this investigation, would not Debye screening of the quark color charge suppress their production? Matsui and Satz [23] have argued that the production of J/Psi should be prohibited when the screening radius is smaller than the radius of the bound state, and that this should happen when the temperature of the plasma is considerably in excess of 200 MeV (which is taken by them to be the deconfinement temperature). In this connection, however, one must also note the findings of Bhatt et al. [24], which are: (a) If a test source is at rest in the plasma, the screening does not depend upon the color dynamics; and (b) For a test source moving with non-relativistic velocity, the non-abelian features manifest themselves by actually *weakening* the screening. In any case, Tu – the temperature in our study which marks the threshold above which the mesons have unstable masses – is well below 200 MeV.

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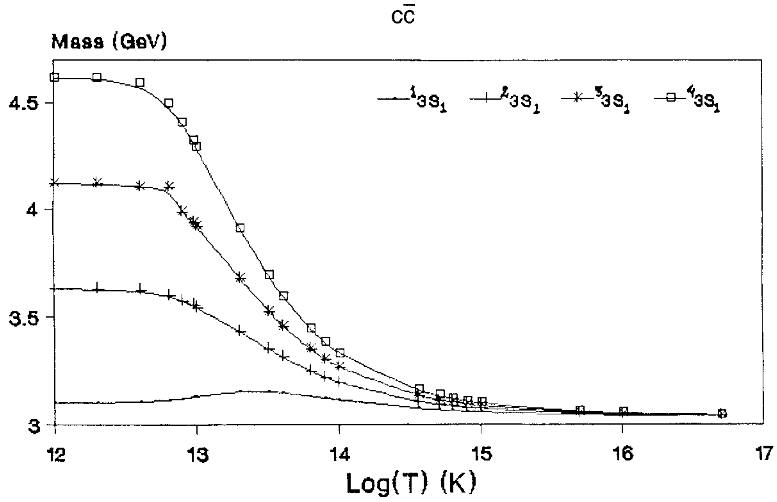

Fig. 2. Variation of theoretical mass of n^3S_1 states of $c\bar{c}$ with logarithm of temperature

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